EE 230

Lecture 3

Background Materials
Transfer Functions

Quiz 2

There are 4 basic ways for representing a time-domain analog signal. What are they?

And the number is ?

1 3 8

5 4

?

2 6

9 7

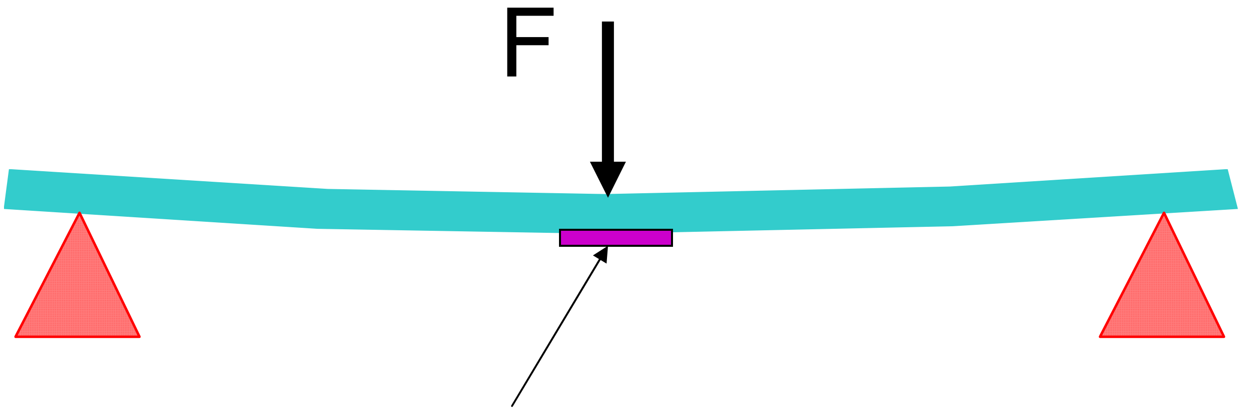
Quiz 2

There are 4 basic ways for representing a time-domain analog signal. What are they?

Laboratory and Class Issues

- Monday lab will "catch up" during week 3 or week 4. Will do experiment 2 during week 3.
- Help with operation of equipment
- 11:00 Lecture in Rm 1014 or 1016 Coover
- Please bring bound notebooks to lab starting for Week 2
- HW 2 will be due on Friday of next week

Review from Last Time



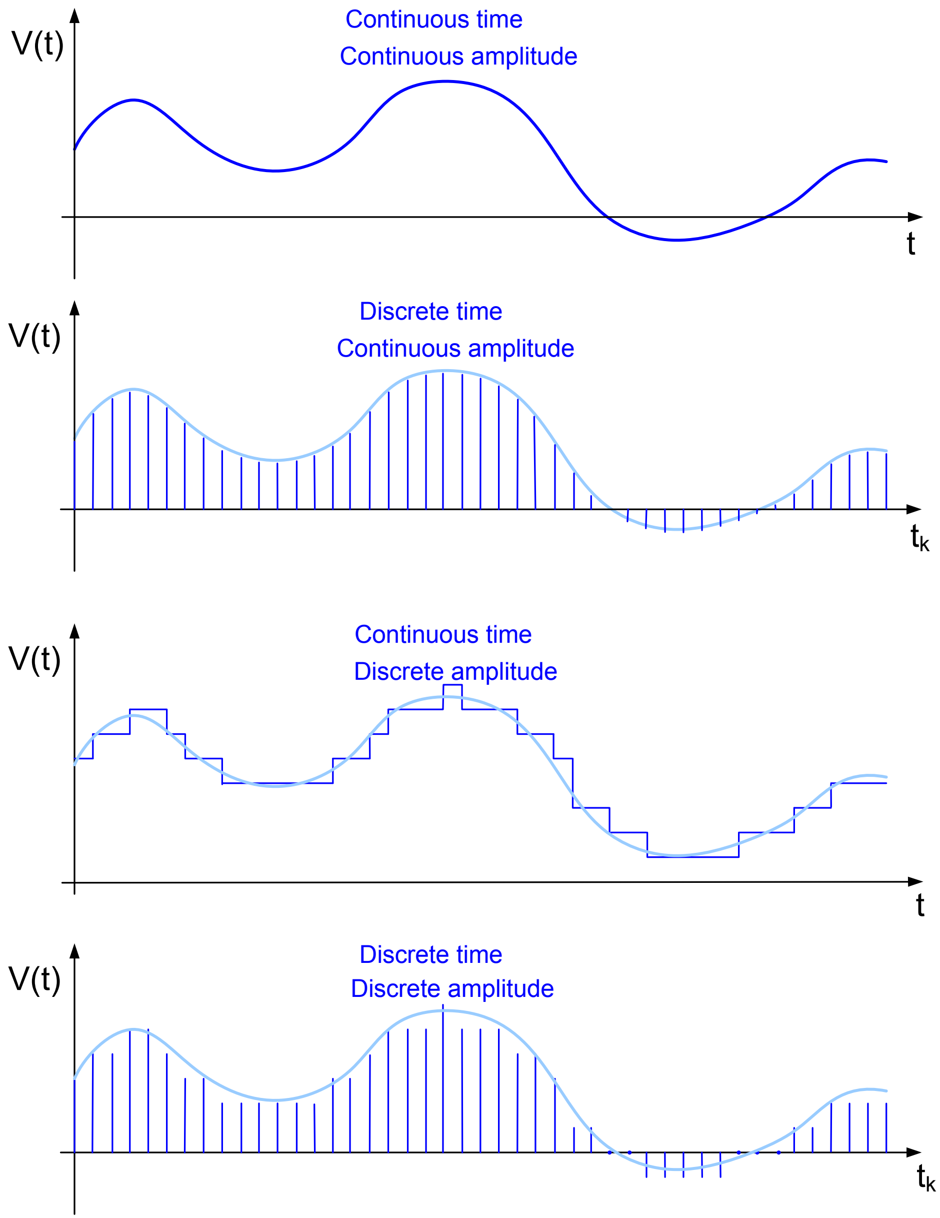
Strain gauge characterization

$$GF = \frac{\Delta R / R}{\Delta L / L} = \frac{\Delta R / R}{\varepsilon}$$

Typical GF for foil strain gauges are around 2

Often but not always represent the same analog CD/CA Analog Signal

Review from Last Time



Review from Last Time

Key property of many useful signals:

Theorem: If f(t) is periodic with period T, then f(t) can be expresses as

$$f(t) = \sum_{k=0}^{\infty} A_k \sin(k\omega t + \theta_k)$$

where A_k and θ_k are constants and $\omega = \frac{2\pi}{T} = 2\pi f$

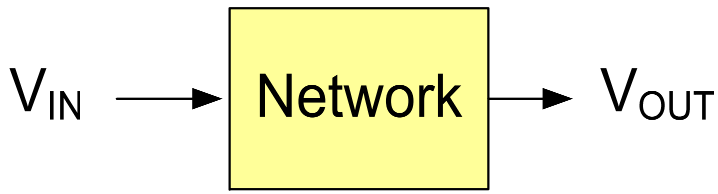
This is termed the Fourier Series Representation of f(t)

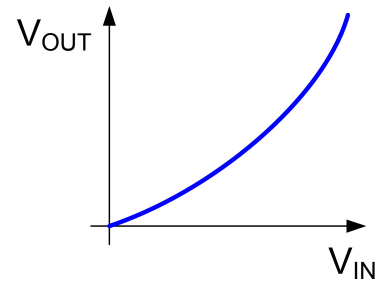
$< A_k, \theta_k >_{k=0}^{\infty} = F(\omega)$ termed the frequency spectrum of f(t)

F(ω) is a vector sequence

f(t) $\longleftrightarrow$ F(ω) represent a transform pair

Linearity





dc transfer characteristics

Definition:

A network is linear if

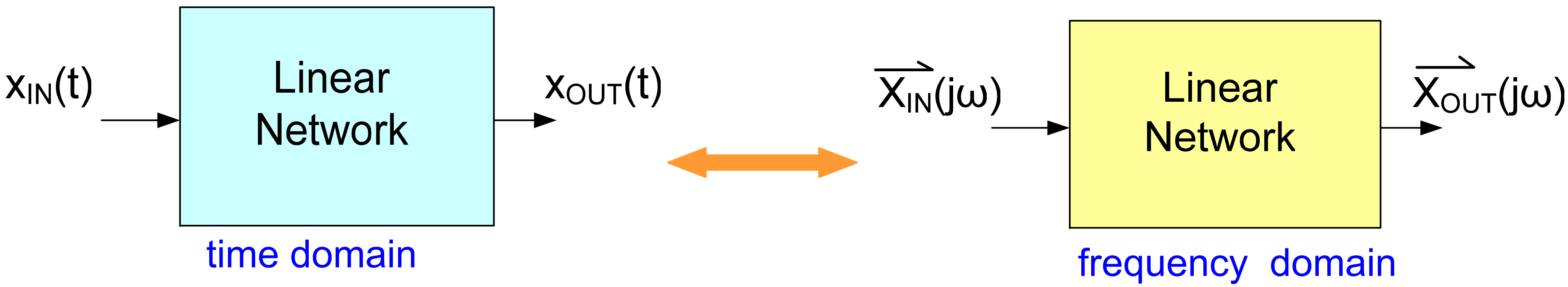
$$V_{OUT}(a_1V_{IN1}+a_2V_{IN2})=a_1V_{OUT}(V_{IN1})+a_2V_{OUT}(V_{IN2})$$

for all constants a_1 and a_2 and for any inputs V_{IN1} and V_{IN2}

- It follows that superposition can be used to analyze a linear network
- If a network is linear, the dc transfer characteristics is a straight line
- If the dc transfer characteristics of a network is not a straight line, the network is nonlinear

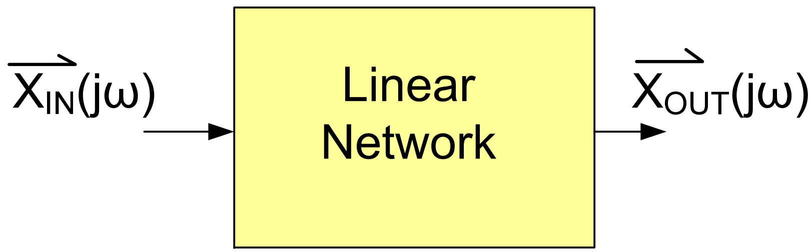
(the linearity definition and properties discussed here and on subsequent slides apply to entities that are referred to by several different names including *circuits, systems, networks, structures, architectures*,…)

Properties of Linear Networks



- A linear network always operates in the time domain
- Time domain and frequency domain representations often used to characterize a linear network
- Mapping between time domain and given frequency domain representation of a given network is unique
- Frequency domain representation often used to analyze or visualize how small sinusoidal signals propagate in the network
- Whether time domain or frequency domain characterization is being considered is determined by context

Properties of Linear Networks



frequency domain

$$\frac{\vec{X}_{OUT}(j\omega)}{\vec{X}_{IN}(j\omega)} = T_P(j\omega)$$

$T_P(j\omega)$ is termed the phasor transfer function

$$T_P(j\omega) = \left|T_P(j\omega)\right| e^{j\angle T(j\omega)}$$

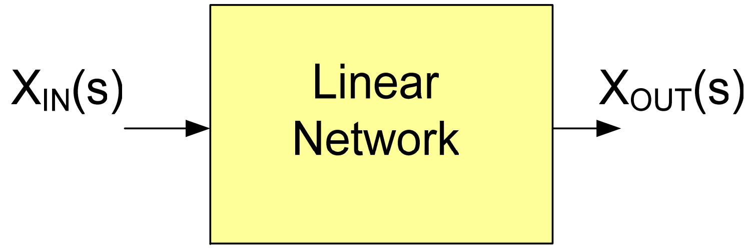
often equivalently expressed as

$$T_P(j\omega) = \left|T_P(j\omega)\right| e^{j\theta}$$

alternate notation of complex quantities

$$\theta = \angle T(j\omega) = \arg\left(T(j\omega)\right) = \tan^{-1}\left(\frac{\operatorname{Im}\left(T(j\omega)\right)}{\operatorname{Re}\left(T(j\omega)\right)}\right)$$

Properties of Linear Networks



frequency domain

$$\frac{X_{OUT}(s)}{X_{IN}(s)} = T(s)$$

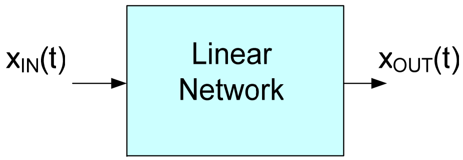
$T(s)$ is termed the transfer function

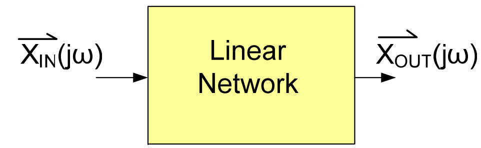
This is often termed the "s-domain" or "Laplace-domain" representation

$$T(s)\big|_{s=j\omega} = T_P(j\omega)$$

Will discuss the frequency domain representations and the more general concept of transfer functions in more detail later

Properties of Linear Networks





If a sinusoidal signal is input to a linear network, no harmonics are present in the output

If a sinusoidal signal is input to a nonlinear network, harmonics often appear in the output

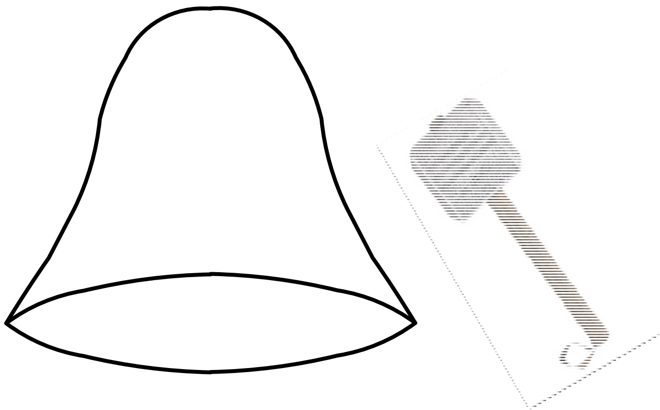
If a sinusoidal signal is input to a network and harmonics appear in the output, the network is nonlinear

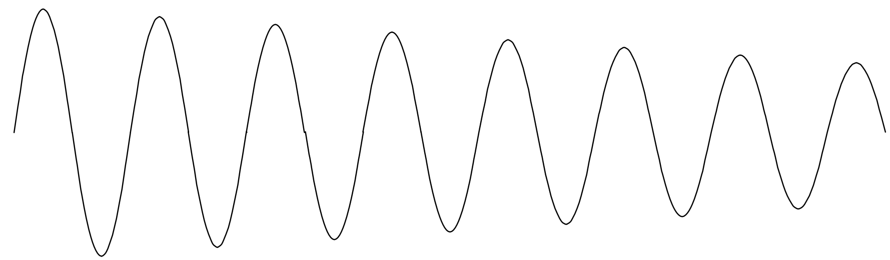
The introduction of harmonics by a nonlinear network creates “distortion” and even very small amounts of distortion are highly undesirable in many systems that are ideally linear

In some nonlinear systems, distortion is desired (but often very particular about type and amount)

A network can behave linearly if the magnitudes of the input signals are not too large but nonlinearly if the input signals are too big

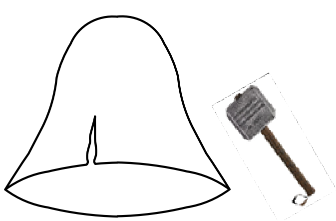
Example:

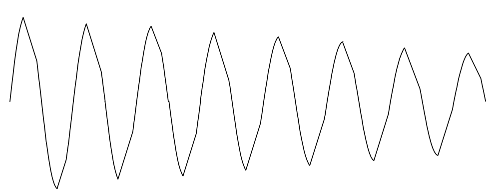




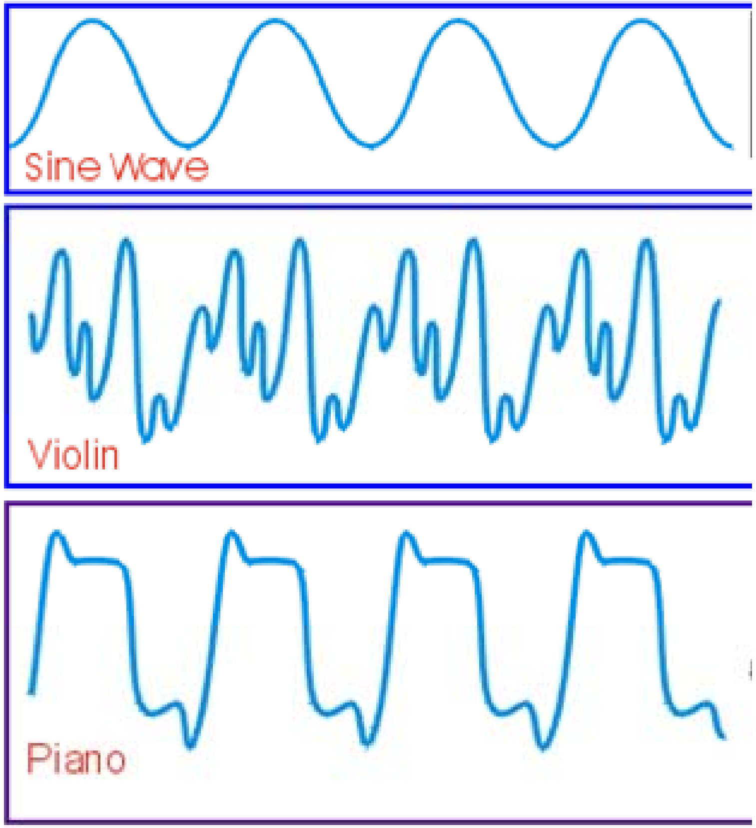
Striking the bell results in a nearly pure sinusoidal waveform that sounds "pleasurable" for a while

If the sinusoidal output were modified by an amplifier or by a defect in the bell, the sound would likely be very disturbing





Example: When distortion is desired

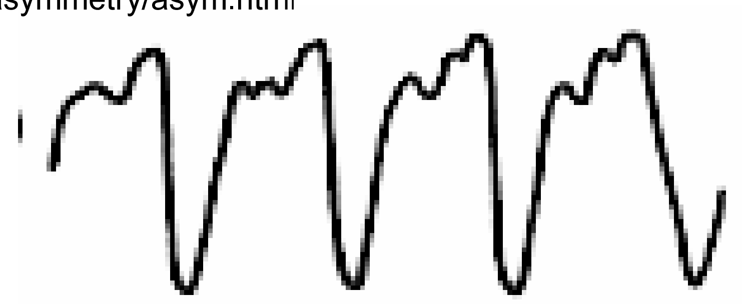


http://method-behind-the-music.com/mechanics/physics

In audio, pure sinusoids become very annoying after a short time

Example: When distortion is desired

http://www.audiomisc.co.uk/asymmetry/asym.html



French Horn

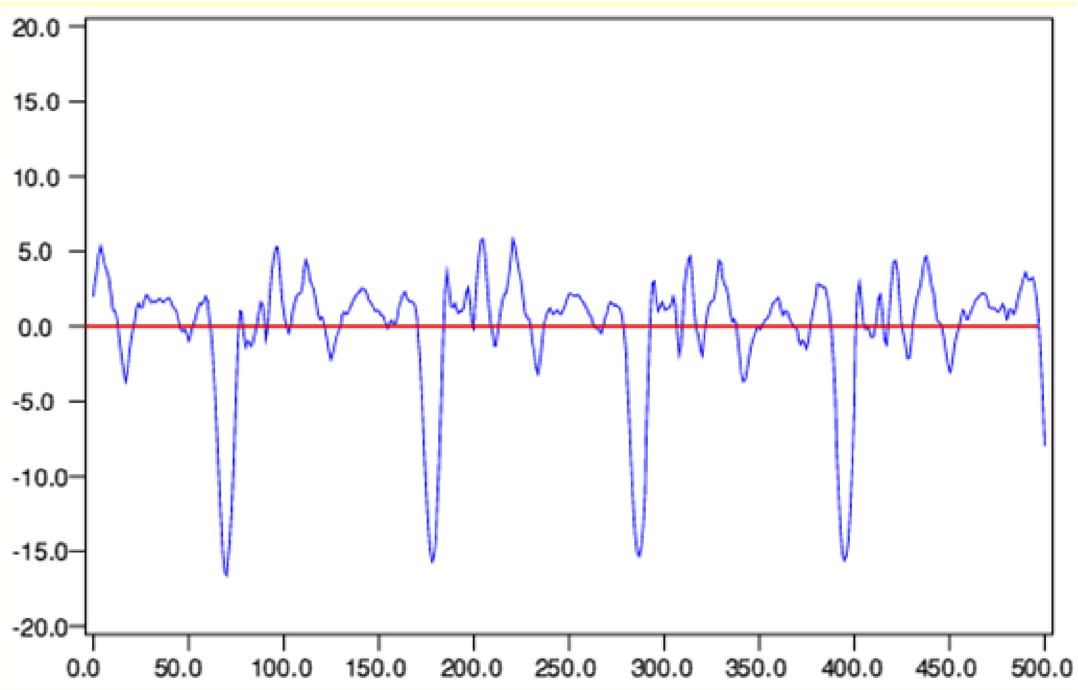
Clarinet

Violin

- Nearly periodic
- Quality of sound strongly dependent upon specific type of distortion

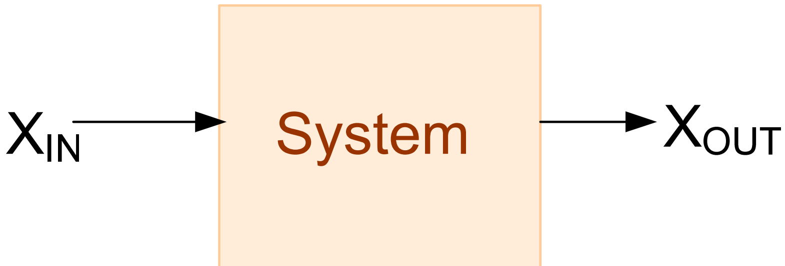
Example: When distortion is desired

http://www.audiomisc.co.uk/asymmetry/asym.html



Trumpet

Distortion



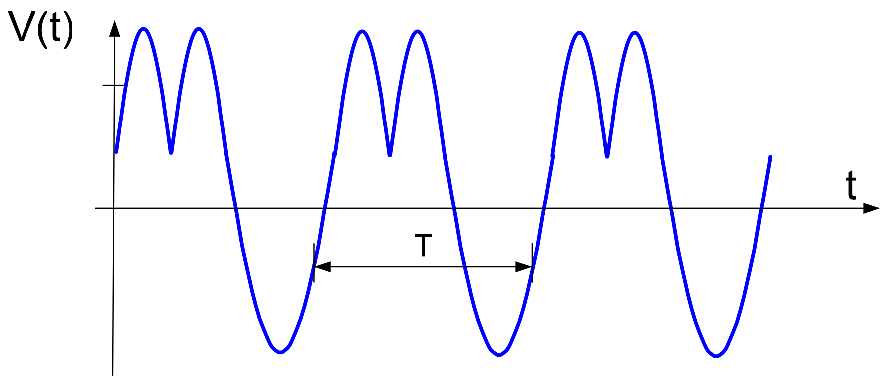
A system has Harmonic Distortion (often just termed “Distortion”) if when a pure sinusoidal input is applied, the Fourier Series representation of the output contains one or more terms at frequencies different than the input frequency

A linear system has Frequency Distortion if for any two sinusoidal inputs of magnitude X_1 and X_2, the ratio of the corresponding sinusoidal outputs is not equal to X_1/X_2.

Harmonic distortion is characterized by several different metrics including the Total Harmonic Distortion, Spurious Free Dynamic Range (SFDR)

Frequency distortion is characterized by the transfer function, T(s), of the system

Total Harmonic Distortion



The Total Harmonic Distortion (THD) is a measure of how much power is in the distortion components relative to the power in the fundamental

Consider a periodic function with zero average value

$$f(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

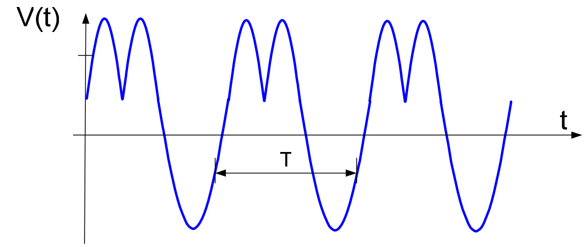
If f(t) is a voltage driving a resistive 1Ω load, then

$$P(t) = f^2(t)$$

$$P_{AVG} = \frac{1}{T} \int_{t_1}^{t_1+T} f^2(t)\,dt$$

Total Harmonic Distortion

$$P_{AVG} = \frac{1}{T}\int_{t_1}^{t_1+T} f^2(t)\,dt$$



$$f(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t+\theta_k)$$

It can be shown that

$$P_{AVG} = \frac{\sum_{k=1}^{\infty} A_k^2}{2}$$

Define P_1 to be the power in the fundamental

$$P_1 = \frac{A_1^2}{2}$$

$$P_{Harmonics} = \frac{\sum_{k=2}^{\infty} A_k^2}{2}$$

$$THD = \frac{P_{HARMONICS}}{P_1}$$

$$THD = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2}$$

THD often expressed in dB or in %

$$THD_{dB} = 10\log_{10}(THD)$$

Can also be expressed relative to signal instead of power

Total harmonic distortion

From Wikipedia, the free encyclopedia

The **total harmonic distortion**, or **THD**, of a signal is a measurement of the harmonic distortion present and is defined as the ratio of the sum of the powers of all harmonic components to the power of the fundamental.

Explanation

In most cases, the transfer function of a system is linear and time-invariant. When a signal passes through a non-linear device, additional content is added at the harmonics of the original frequencies. THD is a measurement of the extent of that distortion.

The measurement is most commonly the ratio of the sum of the powers of all harmonic frequencies *above* the fundamental frequency to the power of the fundamental:

$$\mathrm{THD} = \frac{\sum \text{harmonic powers}}{\text{fundamental frequency power}} = \frac{P_2 + P_3 + P_4 + \cdots + P_n}{P_1}$$

Other calculations for amplitudes, voltages, currents, and so forth are equivalent. For a voltage signal, for instance, the ratio of RMS voltages is equivalent to the power ratio:

$$\mathrm{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \cdots + V_n^2}}{V_1}$$

In this calculation, V_n means the RMS voltage of harmonic n.

Other definitions may be used. A measurement must specify how it was measured. Measurements for calculating the THD are made at the output of a device under specified conditions. The THD is usually expressed in percent as distortion factor or in dB as distortion attenuation. A meaningful measurement must include the number of harmonics included (and *should* include other information about the test conditions).

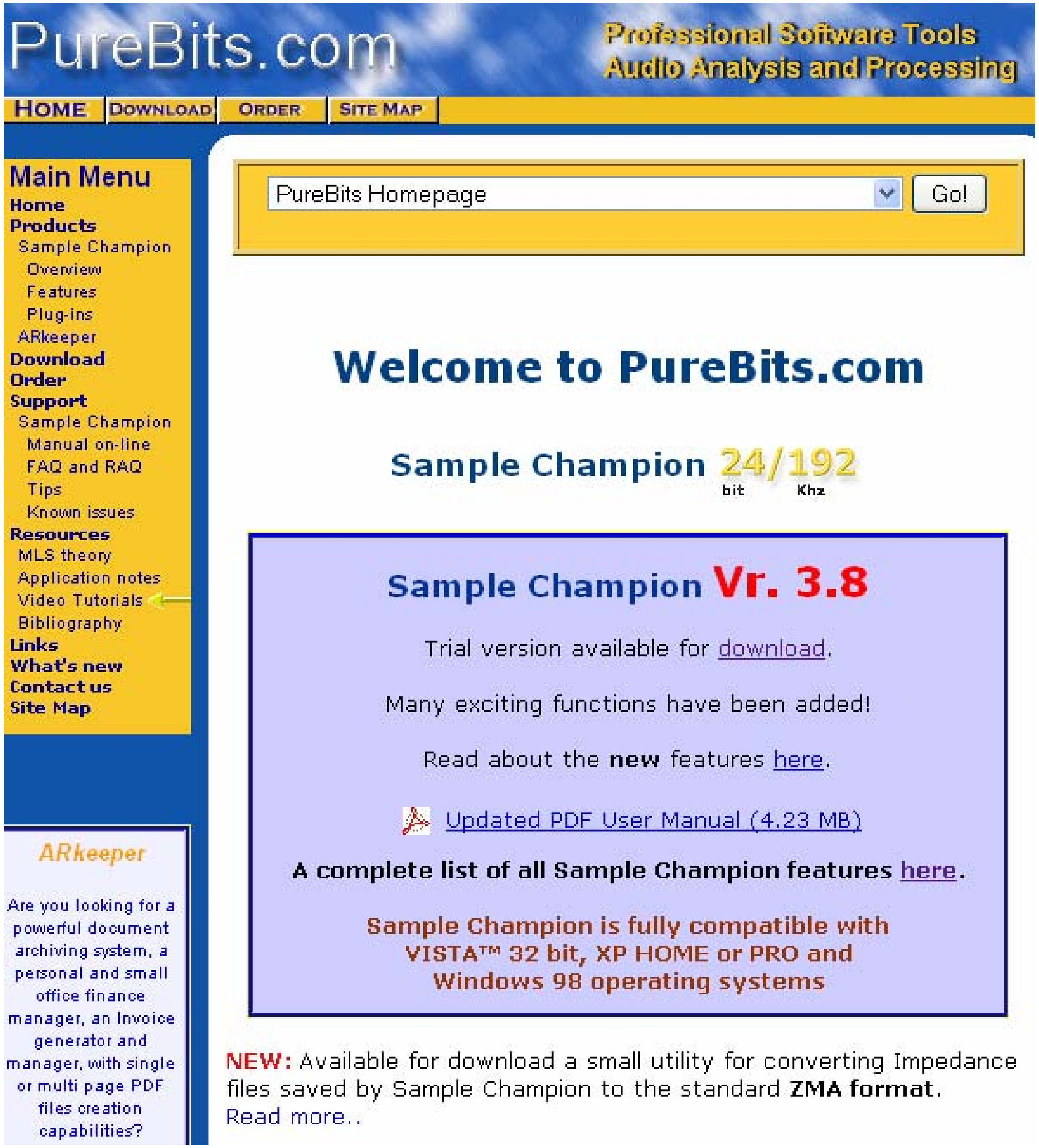
THD+N means total harmonic distortion plus noise. This measurement is much more common and more comparable between devices. This is usually measured by inputting a sine wave, notch filtering it, and measuring the ratio between the signal with and without the sine wave:

$$\mathrm{THD{+}N} = \frac{\sum \text{harmonic powers} + \text{noise power}}{\text{total output power}}$$

A meaningful measurement must include the bandwidth of measurement. This measurement includes effects from intermodulation distortion, interference, and so on, instead of just harmonic distortion.

Signal analysis tool that may be useful

30-day free trial

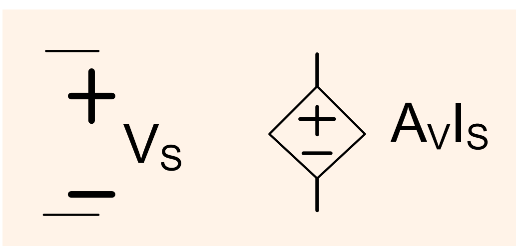


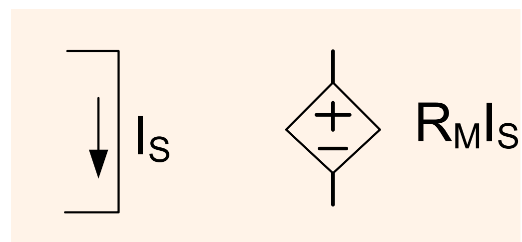
Amplifiers:

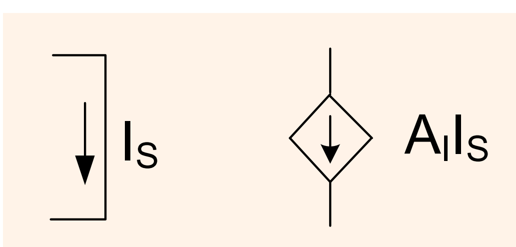
Amplifiers are circuits that scale a signal by a constant amount

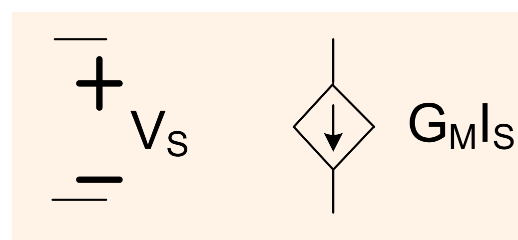
Ideally $V_{OUT}=AV_{IN}$ where A is a constant (termed the gain)

The dependent sources discussed in EE 201 are amplifiers









Amplifiers:

Amplifiers are circuits that scale a signal by a constant amount

$$V_{OUT}=AV_{IN}$$

- The scaling constant is often larger than 1 (when dimensionless)
- For the output to be a scaled version of the input, linearity is assumed
- Linearity is important in most amplifier applications
- Even small amounts of distortion are objectionable in most applications
- Power amplification can be provided in many amplifiers
- Frequency distortion is characterized by a frequency-dependent gain (will be rigorous later)
- Frequency distortion also problematic in many applications
- Frequency distortion can be (and often is) present in linear amplifiers